



Questions 1 and 2 are gridded response items that require you to write your answers in the boxes provided on your answer sheet. Write only one number or symbol in each box and fill in the circle in each column that matches what you have printed. Fill in only one circle in each column.

1 What is the remainder when $x^3 - 1$ is divided by $(x + 2)$?

(-9)
$$\begin{array}{r} -2 \overline{) 1 \ 0 \ 0 \ -1} \\ \underline{-2 \ 4 \ -8} \\ \end{array}$$

$x^2 - 2x + 4 - \frac{9}{x+2}$

2 The function below, $f(x)$, has $(x - 7)$ and $(x + 4)$ as factors.

$f(x) = 2x^3 - 13x^2 + 22x^3 - 187x^2 - 160x + 336$

X-intercepts

$x - 7 = 0 \implies x + 4i = 0 \implies x - 4i = 0$

$x = 7$ (boxed: real)
 $x = -4i$ (boxed: non-real)
 $x = 4i$ (boxed: non-real)

→ there should be 5 roots total

→ graph if you see three real roots where the graph crosses the x-axis



2018-2019 Key

3 The graph of the function $m(x) = x^3 + 3x^2 - 2x - 4$ has a zero at -1. What are the other zeros of the function?

(-2 and 2)
 (-1 and 4)
 (-1 ± √5)
 (1 ± 2√5)

$$\begin{array}{r} -1 \overline{) 1 \ 3 \ -2 \ -4} \\ \underline{-1 \ 2 \ -4} \\ \end{array}$$

$x^2 + 2x - 4 = 0$

4 Which expression is equivalent to $(x^2 - 2x - 37) \div (x^2 - 3x - 40)$?

- A $1 + \frac{x+3}{x^2-3x-40}$
- B $1 - \frac{x+3}{x^2-3x-40}$
- C $1 + \frac{2x-37}{x^2-3x-40}$
- D $1 - \frac{2x-37}{x^2-3x-40}$

$$\begin{array}{r} x^2 - 3x - 40 \overline{) x^2 - 2x - 37} \\ \underline{-(x^2 - 3x - 40)} \\ \end{array}$$

$x + 3$

$1 + \frac{x+3}{x^2-3x-40}$



$a=1 \quad b=2 \quad c=-4$

$$2(1) \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2}$$

$$\frac{-2 \pm \sqrt{20}}{2}$$

$$\frac{-2 \pm 2\sqrt{5}}{2}$$

$$-1 \pm \sqrt{5}$$



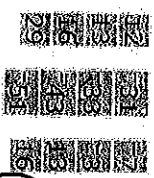
5 This is a paper/pencil copy of an online technology enhanced item.

Place (click and drag) into the appropriate boxes the values of A, B, and C that will make the equation shown below true.

$$\frac{(7x-2)(3x-5)}{(7x-2)} + \frac{-2x+1}{7x-2} = \frac{Ax^2+Bx+C}{(7x-2)}$$

A = B = C =

1 2 3



$$\frac{(7x-2)(3x-5)}{7x-2} + \frac{-2x+1}{7x-2} = \frac{(7x-2)(4x+3)}{7x-2}$$

$$(7x-2)(3x-5) + (-2x+1)(4x+3)$$

$$= 21x^2 - 35x - 6x + 10 - 8x^2 - 6x + 4x + 3$$

$$= 13x^2 - 43x + 13$$

#1 A = 13

#2 B = -43

#3 C = 13

Go to the next page



6 Which expression is equivalent to $\frac{(x^2 - 5x + 6)^{-1}}{(x-2)^2} \div \frac{(x-3)^{-1}}{(x-2)^2}$?

A $\frac{(x-2)^2}{(x-3)^2}$

B $\frac{(x+2)^3}{(x+3)^2}$

C $\frac{1}{x-3}$

D $\frac{1}{x-2}$

$$\frac{(x-2)(x-2)}{(x-2)(x-3)} \cdot \frac{x-3}{(x-2)(x-2)}$$

$$= \frac{1}{x-2}$$

7 A company makes and boxes spaghetti.

- One machine fills each box with approximately 32 ounces of spaghetti.
- After the boxes are filled, another machine weighs each box.
- A box is discarded if the weight of the box differs by more than 0.25 ounce from the target weight of 32 ounces.

Which inequality can be used to find the range of acceptable weights, x , of the spaghetti?

- A $|x - 0.25| \leq 32$
- B $|x + 0.25| \leq 32$
- C $|x - 32| \leq 0.25$
- D $|x + 32| \leq 0.25$

$$-0.25 \leq x - 32 \leq 0.25$$

$$+32 \quad +32 \quad +32$$

32, 36
A, B
are out

$$31.75 \leq x \leq 32.25$$

which makes sense

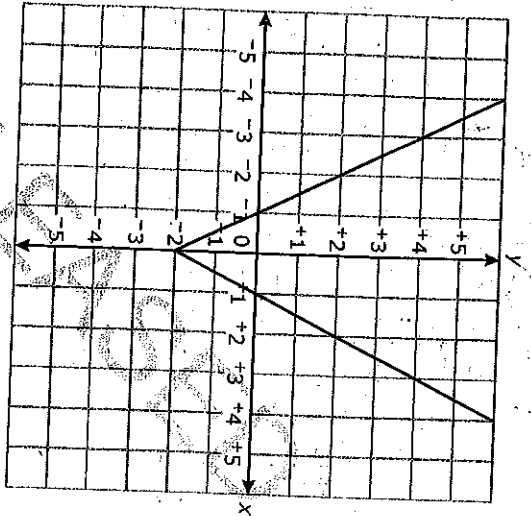
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if $|x| < a$
then $-a < x < a$

OR
if $|x| > a$
then $x < -a$ or $x > a$



8 The graph of an equation is shown below.



Which equation represents the graph?

- A $y = |x| - 2$
- B $y = |2x| - 2$
- C $y = |x - 2|$
- D $y = |2x - 2|$

Plug & Chug

Math \rightarrow Num \rightarrow # 1



Questions 9 through 10 are gridded response items that require you to write your answers in the boxes provided on your answer sheet. Write only one number or symbol in each box and fill in the circle in each column that matches what you have printed. Fill in only one circle in each column.

9 Martha can paint a room in 2 hours. Jamie can paint the same room in 6 hours. How long, to the nearest tenth of an hour, will it take them to paint the room together?

$$\frac{1}{2} + \frac{1}{6} = \frac{1}{x}$$

10 Two functions are shown below.

$$f(x) = 2x^2 + 2x - 3$$

$$g(x) = -0.5|x - 4|$$

What is the y-value when $f(x) = g(x)$?

intersection

Put both functions in the calculator & press

2nd TRACE #5 & look at the y-value

$$y = -1.75$$



$\frac{1}{1st} + \frac{1}{2nd} = \frac{1}{together}$

$$x = 1.5$$



11 Which choice is equivalent to the expression shown below?

- A $3(4x^2 - 9y)(4x^2 - 9y)$
 B $3(4x^2 - 9y)(4x^2 + 9y)$
 C $3x(4x - 9y)(4x - 9y)$
 D $3x(4x - 9y)(4x + 9y)$
- $48x^3 - 243xy^2$ GCF: $3x$
 difference of perfect squares
 $3x(16x^2 - 81y^2)$
 $3x(4x - 9y)(4x + 9y)$

12 A polynomial, $p(x)$, has a lead coefficient of 1 and exactly three distinct zeros.

- $x = -1$ is a zero of multiplicity two
 - $x = 2$ is a zero of multiplicity one
 - $x = 4$ is a zero of multiplicity one
- What choice shows $p(x)$?
- A $p(x) = x^3 - 5x^2 + 2x + 8$
 B $p(x) = x^3 + 5x^2 + 2x - 8$
 C $p(x) = x^4 - 4x^3 - 3x^2 + 10x + 8$
 D $p(x) = x^4 + 4x^3 - 3x^2 - 10x + 8$

multiplicity tells you the exponent, so you have x^4 , A & B are gone!

$x = -1$ $x = -1$ $x = 2$ $x = 4$
 $(x+1) = 0$ $(x+1) = 0$ $(x-2) = 0$ $(x-4) = 0$

$(x+1)(x+1)(x-2)(x-4)$

$(x^2 + 2x + 1)(x^2 - 6x + 8)$

USE BOX METHOD or distribute

x^2	$-6x$	8
x	$-10x^3$	$8x^2$
1	$-12x^2$	$10x$
	$10x$	8

14 Two piecewise functions are shown below.

$h(2) = 4(2) + 1 = 9$ $h(x) = \begin{cases} -3x & \text{for } x < 2 \\ 4x + 1 & \text{for } x \geq 2 \end{cases}$
 $g(1) = 1^2 + 2 = 3$ $g(x) = \begin{cases} x^2 + 2 & \text{for } x < 3 \\ x^3 & \text{for } x \geq 3 \end{cases}$
 ← 2 fits here ← 1 fits here

- A 39
 B 28
 C 10
 D -6

$3(9) + 4(3) = 27 + 12 = 39$

in front > 1 means vertical stretch
 (transformation rules)

Go to the next page

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Question 15 is a gridded/response item that requires you to write your answer in the boxes provided on your answer sheet. Write only one number or symbol in each box and fill in the circle in each column that matches what you have printed. Fill in only one circle in each column.

15 A function is shown below.

$$h(x) = \begin{cases} \frac{1}{2}x - 15 & \text{for } x \leq -4 \\ 20 - 3x^2 & \text{for } x > -4 \end{cases}$$

What is the value of $h(-4) + 3h(-2)$?

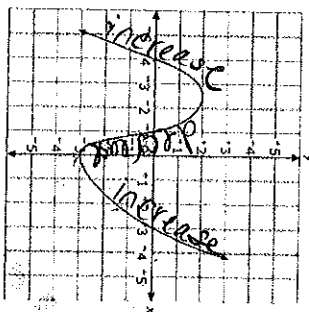
$h(-4) = -\frac{1}{2}(-4) - 15 = -13$

$3 \cdot h(-2) = 3 \cdot (20 - 3(-2)^2) = \frac{24}{11}$

← 4 fits here
← 2 fits here



16 This is a paper/pencil copy of an online technology enhanced item. The graph of a function is shown below.



Positive (above x-axis)

Negative (below x-axis)

Place (click and drag) each interval into the column that describes the function on that interval.

Decreasing	Increasing	Positive	Negative
<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>



Question 17 is a gridded response item that requires you to write your answer in the boxes provided on your answer sheet. Write only one number or symbol in each box and fill in the circle in each column that matches what you have printed. Fill in only one circle in each column.

17 A function is shown below.

$$H(x) = 4x^3 - 5x^2 - 23x + 6$$

What is the distance, to the nearest hundredth of a unit, between the two zeros that are closest to each other?

Handwritten work for Question 17:

$$(-2, 0) \quad X = -2 \quad X = \frac{1}{4} \quad X = 3$$

8.25 away 2.75 away

2.25



18 Which function does *not* have the set of all real numbers as its domain?

- A $f(x) = 5^x - 3$
 - B $f(x) = \frac{x+1}{x+3}$
 - C $f(x) = |2x - 1|$
 - D $f(x) = \cos(x) + 1$
- Handwritten notes: $(-\infty, \infty)$ with an arrow pointing to B; $(-\infty, \infty)$ next to C.

19 An equation is shown below.

$$9^{-3x+2} = 48$$

What is the value of x to the nearest ten-thousandth?

- A 0.0794
- B 0.0995
- C 0.4243
- D 0.4774

Handwritten work for Question 19:

$$\log_9 A = E$$

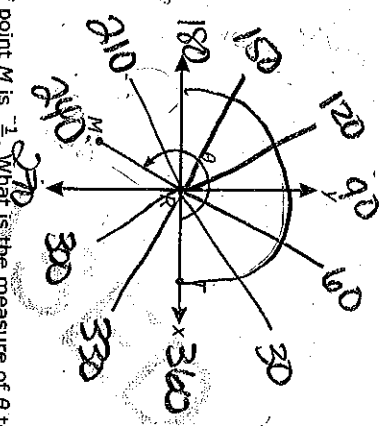
$$\log_9 48 = -3x + 2$$

$$\log_9 48 - 2 = \frac{-3x}{-3}$$

$$X = 0.0794$$



20 The diagram below shows an angle, θ , graphed in the xy -coordinate plane. Segment RT is the initial side of the angle, and segment RM is the terminal side. Segments RT and RM are radii of the unit circle centered at the origin $R(0, 0)$.



The x -coordinate of point M is $-\frac{1}{2}$. What is the measure of θ to the nearest thousandth of a radian?

- A 4.712
- B 4.189
- C 3.927
- D 3.665

Convert to radians

skip

$$\frac{240 \cdot \pi}{180} = 4.1888$$



21 This is a paper/pencil copy of an online technology enhanced item.

amplitude \rightarrow if they gave you a max? min \rightarrow state the average

A function, $f(x) = A \sin(Bx) + H$, has the following properties:

- a period of 6,
- a minimum value of 2,
- $f(2.5) = 5$, and
- A , B , and H are all positive constants.

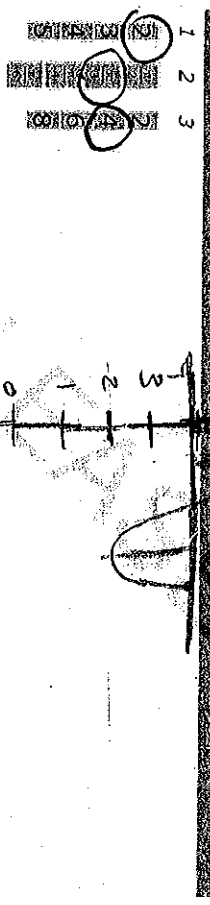
Place (click and drag) values into the appropriate cells below that will create this function.

frequency

$$b = \frac{2\pi}{6}$$

$$b = \frac{2\pi}{6} = \frac{\pi}{3}$$

A = 1 B = 2 H = 3



$$f(x) = \boxed{2} \sin\left(\frac{\pi}{3}x\right) + \boxed{4}$$

\uparrow
if the amplitude is 2
then $2+2=4$
= midline
 $\frac{1}{2} \cdot 4 + 2 = \text{max}$



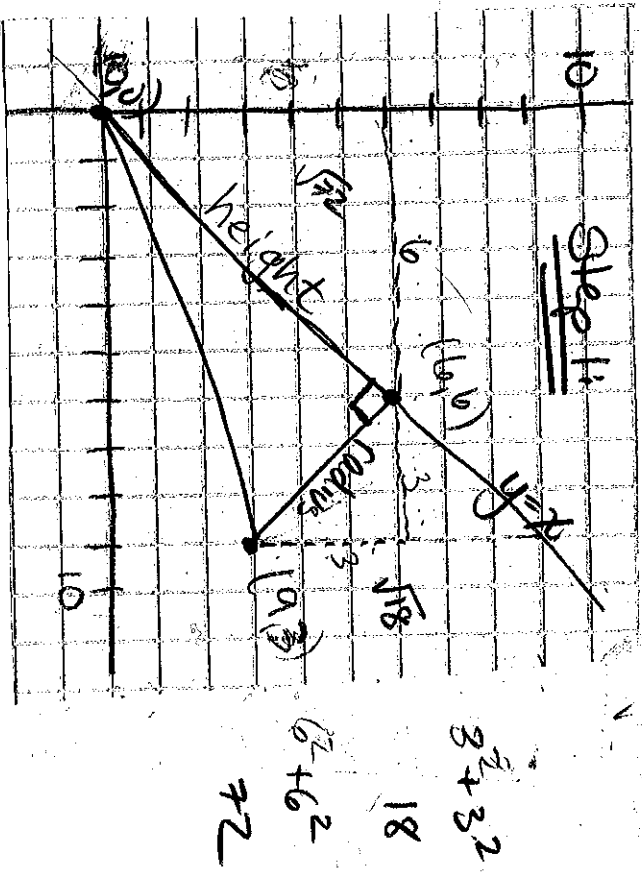


22

The vertices of a triangle are at (0, 0), (6, 6), and (9, 3). What is the volume, in cubic units, of the figure created by rotating the triangle about $y = x$?

- A $54\sqrt{2}$
- B $324\sqrt{2}$
- C $36\pi\sqrt{2}$
- D $108\pi\sqrt{2}$

Step 3 $V = \frac{1}{3}\pi r^2 h$
 $= \frac{1}{3}\pi (\sqrt{18})^2 (8\sqrt{2})$
 $= 36\pi\sqrt{2}$



Step 2
 radius: $\sqrt{18}$
 $(6,6) \rightarrow (9,3)$

$$d = \sqrt{(9-6)^2 + (3-6)^2}$$

$$= \sqrt{9+9}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

height: $8\sqrt{2}$
 $(0,0) \rightarrow (6,6)$

$$d = \sqrt{(6-0)^2 + (6-0)^2}$$

$$= \sqrt{36+36}$$

$$= \sqrt{72}$$

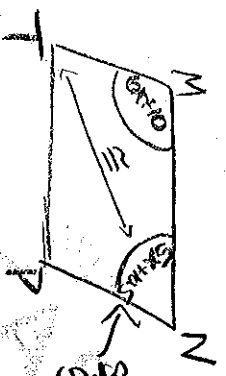
$$= 6\sqrt{2}$$

15 Go to the next page



Question 23 is a gridded response item that requires you to write your answer in the boxes provided on your answer sheet. Write only one number or symbol in each box and fill in the circle in each column that matches what you have printed. Fill in only one circle in each column.

23. In parallelogram $MNPQ$, $m\angle M = (6x + 10)^\circ$ and $m\angle N = (5x + 10.5)^\circ$. How many degrees is $\angle T$?



these angles are supplementary

Step 2
 $5(14.5) + 10.5 = 83$

Step 1:
 $6x + 10 + 5x + 10.5 = 180$

$11x + 20.5 = 180$

$11x = \frac{159.5}{11}$

$x = 14.5$

16 Go to the next page

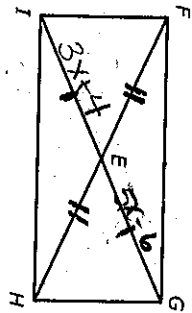
Step 2: Plug x into $\angle N$. since $\angle N \cong \angle T$

$5(14.5) + 10.5$

$\angle N \cong \angle T = 83^\circ$



24 In rectangle $Fghi$, diagonals \overline{FH} and \overline{GI} intersect at E .



- $IE = 3x + 4$
- $EG = 5x - 6$

What is the length of \overline{FH} ?

- A 5 units
- B 10 units
- C 19 units
- D 38 units

diagonals are \cong in total length

Step 1:
 $3x + 4 = 5x - 6$

$4 = 2x - 6$

$10 = 2x$
 $5 = x$

Step 2: $IE = 3(5) + 4 = 19$

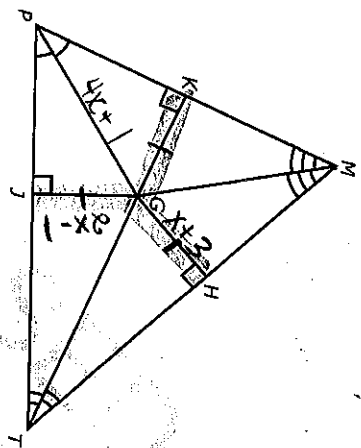
$\frac{38}{x2}$ full length

17 Go to the next page



25 Triangle PMT is shown below.

incenter



- $m\overline{GH} = x + 3$
- $m\overline{PG} = 4x + 1$
- $m\overline{GT} = 2x - 1$

What is the measure of segment \overline{PG} ?

- A 4 units
- B 7 units
- C 14 units
- D 17 units

Step 1: $x + 3 = 2x - 1$

$3 = x - 1$

$x = 4$

Step 2: $PG = 4x + 1$

$4(4) + 1 = 17$

18 Go to the next page



26 What is the length of a radius of the circle represented by the equation $x^2 + y^2 - 4x - 4y + 4 = 0$?

- A 2 units
- B 4 units
- C 8 units
- D 16 units

$$(x^2 - 4x + 4) + (y^2 - 4y + 4) = 4 + 4 + 4$$

$$(x-2)^2 + (y-2)^2 = 4$$

$\leftarrow r^2$ so $r = 2$

29 This is a paper/pencil copy of an online technology enhanced item.

A high school randomly selected 75 of the 200 seniors at the school to take a sample college entrance exam. The mean grade point average (GPA) of the seniors selected was 2.85, and the standard deviation was 0.4. Select (click) all of the statements that are true.

(Note: Margin of error $\approx 2 \cdot \frac{s}{\sqrt{n}}$, where s is the standard deviation and n is the sample size.)

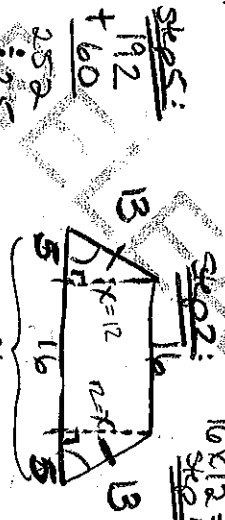
- The margin of error for the mean grade point average is about 0.092.
- The margin of error would decrease if the sample size were changed to 125 seniors.
- The margin of error would increase if the sample size were changed to 125 seniors.

Selected:

Step 27 David plans to cover the floor of his room with new material.
 The floor is an isosceles trapezoid whose bases are 16 feet and 26 feet and sides are 13 feet in length.
 Each piece of new material has an area of 2.5 square feet.

Assuming the pieces of new material can be cut as needed, how many pieces does David need?

- A 101
- B 126
- C 202
- D 252



Step 5: 192
 Step 2: $16 \times 12 = 192$
 Step 1: $5^2 + 12^2 = 13^2$
 $25 + 144 = 169$
 $X^2 = 144$
 $X = 12$

28 Which method is most likely to produce a random sample of 5 students from a school club?

- A selecting 5 club members who have brown hair
- B selecting the 5 club members who have raised the most funds for the club
- C selecting 5 club members from a hat containing the names of all members
- D selecting the 5 club members who arrive last to a club meeting

test for line 3 $\approx 2 \cdot \frac{0.4}{\sqrt{125}} \approx 0.072$ ✓ True
 line 4 would be false

Go to the next page

Go to the next page



30 The police chief of a town is trying to determine the average speed of drivers on a specific road.

- The police chief took a random sample of the speeds of 100 drivers on the road.
- The mean was 48.5 miles per hour, with a standard deviation of 2.5 miles per hour.
- The police chief wanted to decrease the margin of error of his next sample.

Which choice would decrease the margin of error?

(Note: margin of error $\approx 2 \frac{s}{\sqrt{n}}$ where s is the sample standard deviation and n is the sample size.)

- A decreasing the sample size
- B increasing the sample size
- C changing the time that the data is collected
- D changing the road on which the data is collected

Cross out answers that don't make sense

$$\text{margin of error} \approx 2 \frac{2.5}{\sqrt{100}} = 0.5$$

test choice A to any sample size less than 100; I will test 90

$$\text{margin of error} \approx 2 \frac{2.5}{\sqrt{90}} \approx 0.527$$

this increased the margin of error, so answer choice A is out; the answer is B

to test choice B, increase the sample size; I will use 150

$$\approx 2 \frac{2.5}{\sqrt{150}} \approx .408$$

this works!

This is the end of the NC Math 3 released items.

